

be very sensitive to ϵ when this quantity is small, and his results will therefore be strongly dependent on ϵ . A more complicated energy balance equation is required for a realistic estimate of T_{e2} . This must include conduction and convection terms (see, e.g., Grewal and Talbot⁵), but may neglect Q_{e1} because of the assumed small extent of region 1-2.

Another difficulty arises in Truitt's analysis of region 2-3. His ionization rate equation, Eq. (6) of his note, assumes that reaction (2) is the only ionization mechanism. However, this mechanism requires free electrons to be present in significant numbers, and experimental evidence⁴ suggests that, if ϵ_1 is small, the initial part of region 2-3 will be dominated by other mechanisms. Also, Eq. (1) and, hence, Truitt's Eq. (8) cannot be expected to apply in this region, because Q_{e1} is proportional to the rate of reaction (2), which is seen to be small. A more appropriate electron energy equation would express a balance between Q_{e1} and the convective rate of change of electron temperature.

It is only in the final part of zone 2-3, where ϵ has become large enough for reaction (2) to dominate, that Eq. (1) and, hence, Truitt's analysis become accurate. His main conclusion, that T is nearly constant throughout the region 2-3, is likely to be strongly influenced by the effects discussed in this note.

References

- ¹ Truitt, R. W., "Thermal ionization behind strong shock waves," AIAA J 1, 2175-2176 (1963).
- ² Bray, K. N. C. and Wilson, J. A., "A preliminary study of ionic recombination of argon in wind tunnel nozzles, Part II," Univ. Southampton Rept. AASU 185 (1961).
- ³ Bray, K. N. C., "Electron-ion recombination in argon flowing through a supersonic nozzle," *High Temperature Aspects of Hypersonic Flow*, edited by W. C. Nelson (Pergamon Press, Inc., London, 1963).
- ⁴ Petschek, H. and Byron, S., "Approach to equilibrium ionization behind strong shock waves in argon," Ann. Phys. (N.Y.) 1, 270-315 (1957).
- ⁵ Grewal, M. S. and Talbot, L., "Shock-wave structure in a partially ionized gas," J. Fluid Mech. 16, 4 (August 1963).

Reply by Author to K. N. C. Bray

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THE author appreciates the interest of Bray in his earlier paper¹. The purpose of that paper was to present a certain mathematical model which should be regarded as a first approximation to the solution of thermal ionization behind strong shock waves in argon.

Bray correctly describes the normal shock-wave regions usually considered in the strong shock analysis. The author's analysis in Ref. 1 only considers the region in which thermal ionization due to electron-atom collisions occurs behind the strong shock in argon. No attempt is made to analyze the structure in the shock front (region 1-2). Contrary to what Bray said, his Eq. (1) was not used in the region (1-2) because p_2 , ρ_2 , and u_2 can be calculated as a function of λ and ϵ without need to use an energy equation for the electrons. Bray expresses concern over the author's use of a small value of ϵ_1 ($\epsilon_1 = 0.001$) in the initial part of region 2-3 and the assumption of only one ionization mechanism, namely, thermal electron-atom collisions [Eq. (6) of Ref. 1].

It is generally agreed that there are essentially three processes that can produce ionization in a hot gas: 1) thermal

atom-atom collisions, 2) thermal electron-atom collisions, and 3) absorption of radiation from other regions of the hot gas. Of these three, the thermal electron-atom collisions give the best yield, and the process of absorption of radiation is the most unlikely. Since, however, the cold gas upstream of the shock is usually considered initially un-ionized and does not contain electrons, then electrons for the second process must be produced by atom-atom collisions before electron-atom collisions can contribute appreciably to the rate of ionization. The atom-atom reaction is assumed to start immediately downstream of the shock front.

According to Weymann,² it is necessary, therefore, to distinguish between two stages in the ionization. In the initial stage, atom-atom collisions build up the electron and ion density until the electron concentration becomes high enough for the final stage ($\epsilon \approx 0.001$).² In the final stage, electron-atom collisions are the most efficient and account for most of the increase of ionization. As a matter of fact, the ionization rate in the final stage is so very much higher than the initial stage (atom-atom collisions) determines the over-all relaxation time. It was on the basis of Weymann's findings² that the author chose $\epsilon = 0.001$ for the analysis of Ref. 1.

Since, for $\epsilon \approx 0.001$, atom-electron collision is the dominant ionization mechanism, use of Eq. (8) of Ref. 1 is justified if one assumes that $\epsilon \geq 0.001$ at start of region 2. If, however, a value of ϵ much less than this value is chosen at the starting point, other mechanisms should be taken into consideration.

Taking $\epsilon_1 = 0$, the author has carried out a two-stage strong shock analysis (unpublished), using Weymann's theory for the initial atom-atom collisions² to produce an electron concentration such that $\epsilon = 0.001$. It was assumed that at this value of ϵ the electron concentration is high enough for the final stage (the electron-atom ionization mechanism) to commence.

The results of the author's two-stage analysis were essentially the same as those reported in Ref. 1, as regards the nature of the variation of the two temperatures as they approach equilibrium. The main difference in the results of the two-stage analysis and that of Ref. 1 is that the final stage of ionization was found to occur over a much shorter distance than the initial stage. It was found that there was virtually no change in the shock parameters over the entire atom-atom reaction region.

The author realizes that the two-stage analysis is not entirely satisfactory since, for one thing, an average cross section was assumed for atom-atom collision. What is needed is a complete rate equation that simultaneously takes into account both atom-atom and electron-atom collisions. It appears that Eq. (3.3.5) of Ref. 3 meets these requirements. Sufficient experimental cross-section data are now available for argon to allow an approximate analysis of the thermal ionization behind strong shock waves. The author is presently carrying out a strong shock analysis using this complete rate equation, the results of which will be published at a later date.

The author is fully aware of the many complex aspects of the strong shock problem. It is to be hoped that a relatively simple mathematical model can be found which will depict the essential features of the thermal ionization processes behind strong shock waves in argon.

References

- ¹ Truitt, R. W., "Thermal ionization behind strong shock waves," AIAA J 1, 2175-2176 (1963).
- ² Weymann, H. D., "On the mechanism of thermal ionization behind strong shock waves," The Institute for Fluid Dynamics and Applied Mathematics, Univ. Maryland, TN BN-144, Air Force Office Scientific Research TN-58-788 (July 1958).
- ³ Ludwig, G. and Heil, M., "Boundary-layer theory with dissociation and ionization," *Advances in Applied Mechanics* (Academic Press, New York, 1960), Vol. VI, pp. 39-118.

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